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Parameters of Nocilla Gas/Surface Interaction Model from Measured Accommodation Coefficients

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Free-molecule aerodynamic coefficients are computed for a flat plate surface element at various angles of attack using the normal and tangential accommodation coefficients and the Nocilla model of the gas/surface interaction. The Nocilla model assumes that the molecules reflect from a surface in the form of a drifting Maxwellian with a mean velocity at a specific temperature. The computations use curve fits of the angular variation of the existing measurements of the accommodation coefficients and of the parameters in the Nocilla model. The two methods indicate that a surface element has considerably more lift than predicted by an assumption of diffuse scattering. In other respects the predictions by the two methods do not agree. The accommodation coefficients predicted by the Nocilla model using the measured model parameters do not match the measured accommodation coefficients, and the computed temperature of the reflected distribution function is much too large. Since the Nocilla model parameters are so difficult to measure, it is shown that reasonable values of the Nocilla model parameters can be found using the measured accommodation coefficients. These parameters could then be used to provide the re-emitted molecular velocity distribution function for use with the Monte Carlo computational method, for computing low density vehicle aerodynamics.

Nomenclature

B = given by Eq. (19)
E = energy flux
f = fraction of molecules diffusely scattered; velocity distribution function
j = number of internal degrees of freedom of a molecule
k = Boltzmann constant
m = mass of molecule
n = number density
P_r = parameter in Eq. (10)
p = normal momentum flux
R = gas constant
S = speed ratio
T = temperature
V = macroscopic velocity
α_2 = partial thermal accommodation coefficient
γ = ratio of specific heats
δ = angle of attack
θ = angle measured with respect to the local surface normal
ξ = molecular velocity
ρ = density
Σ = given by Eq. (13)
σ = tangential momentum accommodation coefficient
σ' = normal momentum accommodation coefficient
σ_2' = partial normal momentum coefficient
τ = tangential momentum flux
χ = given by Eq. (14)

Subscripts
i = incident
r = reflected
w = wall

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Introduction

EARLY space flights did not require a good understanding of the aerodynamics of rarefied flow. Notable exceptions were the Gemini and Apollo re-entry vehicles, and the difficulties with their re-entries were attributed to the lack of understanding of their rarefied regime aerodynamics. The first flight of the Space Shuttle emphasized again the need for more understanding of this flight regime.[1] It was shown that a 100% larger than expected body flap deflection was required to maintain the desired trim during the rarefied flow portion of the re-entry trajectory. Fortunately, there was sufficient margin to satisfy the increased demand; however, these margins may be too large to be acceptable for future applications such as Mars entry and return vehicles, the National Aerospace Plane (NASP), or any other vehicle that relies on the aerodynamics of the upper atmosphere to achieve its mission.

Rarefied aerodynamic coefficients are determined by the transfer of momentum and energy during the interaction of incident molecules with the body surface. At the highest altitudes the collisions between the incoming and re-emitted molecules can be neglected, and the regime is called free molecule. At lower altitudes collisions between incoming molecules and exiting molecules as well as between incoming molecules and the surface must be taken into consideration. This regime is called the transition regime. In this regime the details of the gas/surface interaction process are also important, and the aerodynamic coefficients depend intimately on that process. Thus, a knowledge of the interaction of incoming molecules with the surface of a vehicle is important for the accurate prediction of aerodynamic coefficients in the transition and free-molecule flow regimes.

The Monte Carlo computation technique is one of the most useful methods for predicting the aerodynamics of bodies which pass through the transition regime and into the free-molecule regime (see, for example, Dogra et al.[2]). This technique requires knowledge of the velocity distribution function of the molecules which are re-emitted from the surface after they have collided with the surface. Traditionally this interaction has been accounted for by the introduction of accommodation coefficients. These quantities are averages over the re-emitted velocity distribution function and do not provide the statistical information needed by the Monte Carlo method. Recently Hurlbut[3] pointed out the usefulness of the Nocilla model for this purpose. The Nocilla model gives an analytical form to the reflected distribution function and is thus very convenient to use with the Monte Carlo method.

The values of the accommodation coefficients and of the parameters of the Nocilla model that apply to a vehicle in low Earth orbit were estimated, using previously reported measurements. The lift and drag of a flat plate surface element were computed using the two techniques. The computational results were not consistent. The discrepancy between the two predictions will be examined and a set of Nocilla model parameters that are consistent with the measured accommodation coefficients will be given. Use of these Nocilla model parameters in the Monte Carlo method would yield useful aerodynamic results throughout the transition range. These results would be very different from previous published results that have relied upon the assumption of diffuse scattering; in particular, the computations would predict considerably greater lift and a very different pitching moment.

Free-Molecule Boundary Condition

Free-molecule aerodynamic calculations require knowledge of the velocity distribution function of the molecules re-emitted from the surface, given the incident velocity distribution function. Originally Maxwell[4] speculated that a fraction f of the incident molecules would adhere to the surface long enough to come to thermal equilibrium with the surface and would be reflected in a diffuse manner and the remaining fraction $(1 - f)$ would be reflected specularly. This assumed surface interaction model is still used in engineering practice, although it does not accurately describe the actual interaction and yields inaccurate aerodynamic coefficient predictions.

This suggestion by Maxwell was followed by the introduction of three accommodation coefficients which describe the degree of accommodation of the incident normal momentum, tangential momentum, and kinetic energy to those of the surface. The coefficient f introduced by Maxwell is identical to the tangential accommodation coefficient. The accommodation coefficients are statistical averages over the distribution function of the re-emitted molecules and, in usual practice, over the distribution function of the incoming molecules. The averaging removes detailed information about the distribution function of the re-emitted molecules, and knowledge of the accommodation coefficients does not uniquely define the re-emitted distribution function. Experiments indicate that the accommodation coefficients are a function of the incoming molecule/surface pair, the energy of the incoming molecule, and the material, temperature, and condition of the surface. The values are different if the surface is free of absorbed molecules, is microscopically smooth, and possesses an ordered molecular structure than if it is a contaminated engineering surface. In addition, their values are found to depend on the value on the angles of the incident and reflected molecular streams. This latter finding led to the introduction of "partial" accommodation coefficients which could be used to incorporate the angular dependence of the accommodation coefficients.[5] Information about the partial accommodation coefficients has not normally been incorporated into engineering design procedures.

Another approach to the description of the gas/surface interaction has been to develop models of the re-emitted distribution function. The re-emitted distribution function should be connected to the incoming distribution through a joint probability distribution function.[6] The joint probability function is presently unknown and simpler models of the interaction process have been made. The most useful model, by Nocilla,[7] describes the re-emitted distribution function as a "drifting Maxwellian," with several parameters which are a function of the incident conditions as well as the gas/surface molecular pair and surface conditions. This model decouples the re-emitted and incoming distribution functions.

In the following section measurements of the accommodation coefficients for situations of interest to vehicles orbiting the Earth will be reviewed. This will be followed by a discussion of the Nocilla model and a review of the measurements that have been made of the model's parameters.

Momentum Accommodation Coefficient Measurements

The momentum accommodation coefficients are defined by the equations

$$\sigma' = \frac{p_i - p_r}{p_i - p_w} \tag{1}$$

$$\sigma = \frac{\tau_i - \tau_r}{\tau_i} \tag{2}$$

where σ' and σ are the normal and tangential accommodation coefficients, respectively. In these equations p is a normal momentum flux and τ a tangential flux. The subscript i indicates the incident quantity, r the re-emitted and w the corresponding quantity at the wall temperature. In the traditional limits of diffuse reflection, $\sigma' = \sigma = 1$, and of specular reflection, $\sigma' = \sigma = 0$.

Numerous measurements of the momentum accommodation coefficients have been made in the past but very few under the conditions experienced by a vehicle in low Earth orbit. Specifically, we are interested in measurements of the momentum accommodation coefficients of atmospheric gases interacting with low temperature (around 300 K) contaminated glass or oxide-covered polycrystalline metallic surfaces at a relative energy in the range of 5–10 eV. Measurements performed using different gases or single crystal surfaces are not of interest.

The incoming flow toward an element of an orbiting vehicle resembles conditions which would be encountered in a molecular beam experiment, where the incident direction is well defined. In this case, the accommodation coefficients depend on the angle of incidence and angle of reflection, and it is meaningful to use partial accommodation coefficients, as introduced by Hurlbut.[5] The measurements to be used in this work were integrated over all reflected angles, and so, for example, the relevant partial normal momentum coefficient is only a function of the incident angle. The partial normal accommodation coefficient is given by

$$\sigma_2'(\theta_i) = \frac{p_i(\theta_i) - p_r}{p_i(\theta_i) - p_w} \tag{3}$$

In this work θ_i is the angle of the incident flow measured from the surface normal.

Accommodation coefficients are most directly determined by measuring two components of the force which are applied to a surface element by an impinging molecular beam, plus incoming beam properties. The angle between the molecular beam and the surface normal can be varied to yield the angular variation of the partial accommodation coefficients. All of the measurements used for the present work, with one exception noted subsequently, were obtained in this manner. For a complete review of accommodation coefficient measurements see Collins.[8]

Measurements of momentum accommodation coefficients obtained under the conditions of interest are summarized in Figs. 1 and 2. The sources of the measurements are given in Table 1. Knechtel and Pitts[9,10] measured the coefficients for the scattering of Ar^+ and N_2^+ from aluminum in the energy range 9–50 eV; measurements were obtained over the angular range of 0–75 deg, measured from the normal. Mair et al.[11] scattered N_2^+ from copper at normal incidence over the energy

Table 1 Accommodation coefficient measurements

Data Set	Gas	Surface	Energy, eV	Reference
1	Ar^+	Au	15	9
2	Ar^+	Al	15	9
3	N_2^+	Fresh varnish	25	13
4	Ar	Fresh varnish	25	13
5	N_2	Al	25	13
6	Ar	Al	25	13
7	N_2^+	Al	5	10
8	N_2^+	Al	10	10
9	N_2^+	Al	15	10
10	N_2^+	Cu	4.4–11.9	11
11	He	Al	1.0	14
12	Ar^+	Au	25	9
13	Ar^+	Al	25	9
14	N_2^+	Al	20	10
15	N_2	Al, etc.	2.0	12

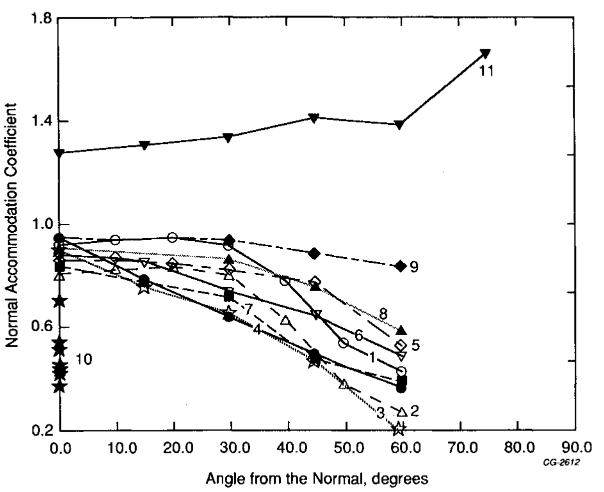

Fig. 1 Normal momentum accommodation coefficient measurements, explanation given in Table 1.

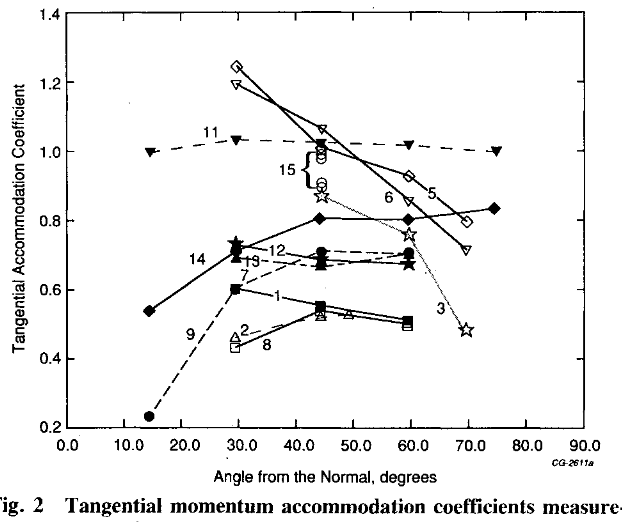

Fig. 2 Tangential momentum accommodation coefficients measurements, explanation given in Table 1.

range 4–100 eV. Musanov et al.[12] scattered N_2 from aluminum, glass, and several other materials at 2 eV. They measured the accommodation coefficients over the angular range of 0–80 deg but only reported values at 45 deg. Doughty and Schaetzle[13] measured the coefficients for neutral air, N_2, and Ar scattered from aluminum and varnish over the energy range of 25–200 eV and the angular range of 0–70 deg. Liu et al.[14] obtained the coefficients for the scattering of He from aluminum at 1 eV, 0–75 deg, by integrating the measured spatial and energy scattering distributions. To the authors' knowledge, this is the only attempt that has even been made to integrate scattered flux distributions to obtain the accommodation coefficients.

In the present application neutral molecule scattering is of primary interest since the atmosphere is composed principally of neutral gases. Knechtel and Pitts[9,10] argued that Auger transitions neutralized the ion before it hits the metal target, and Mair et al.[11] predicted only small differences between the scattering of neutral and ionized nitrogen at the energies of interest. Nevertheless, much of the limited available data is of uncertain value to the present problem since these various arguments concerning the use of ion beams to represent neutral beams have not been tested by direct measurements.

There is general agreement among the various investigators that the normal accommodation coefficient decreases as the incidence angle (measured from the target normal) increases (Fig. 1). The one exception is displayed by the values from Liu et al.[14] which were indirectly determined from scattering flux measurements. This particular data set will be shown to lead to negative lift illustrating how difficult it is to accurately integrate the scattering fluxes. The tangential accommodation coefficient measurements, shown in Fig. 2, do not exhibit a similar agreement between different investigators. Close examination indicates that the measured values have different angular and energy dependencies, according to the experimenter. Most notably, the measurements by Doughty and Schaetzle[13] appear to have a different angular dependence compared to the other measurements. These latter measurements lead to a negative lift coefficient for a flat plate at a large angle of attack, and thus the angular dependence must be questioned.

Seventeen curve-fits were made of the accommodation data shown in Figs. 1 and 2. The complete list of models is given by Knox et al.[15] Only the models listed in Table 2 will be used in the present work. Fit 1 was given by Knechtel and Pitts[17] as a fit to their data at an energy of 10 eV. Fit 14 used a fit to the Doughty and Schaetzle[13] tangential accommodation data. Fit 16 is a fit to the Liu et al.[14] data for helium. For comparison, diffuse scattering is included as model 3. Aerodynamic coefficients obtained using these models will be compared later.

Table 2 Curve-fit equations for accommodation coefficients

Fit 1
Source: Ref. 10

$$\sigma' = 1 - 0.9 \exp[-2.8 \cos^2\theta_i)^2]$$
$$\sigma = 0.9 - 1.2 \exp[-1.47(\sin \theta_i)^{3/4}]$$

Fit 3
Source: diffuse

$$\sigma' = 1.0$$
$$\sigma = 1.0$$

Fit 11
Source: Ref. 10

$$\sigma' = 1 - 0.9 \exp[-2.8 \cos^2\theta_i]$$
$$\sigma = 0.7 \text{ (assumed)}$$

Fit 14
Source: Ref. 10

$$\sigma' = 1 - 0.9 \exp[-2.8 \cos^2\theta_i]$$

Source: fit to Ref. 13 data, N_2/varnish

$$\log_{10}\sigma = \left\{\frac{0.22981 - (\theta_i/90)^2}{1.8081}\right\}$$

Fit 16
Source: fit to He data from Ref. 14

$$\sigma' = 1.28 + 0.1146\theta_i, \qquad \theta_i \text{ radians}$$
$$\sigma = 1.015$$

Nocilla Model

Nocilla[7] proposed that the velocity distribution function of the reflected molecules could be represented by a drifting Maxwellian given by the equation

$$f_r(\xi) = n_r\left(\frac{m}{2\pi kT_r}\right)^{3/2}\exp\left\{-\frac{m(\xi - V_r)^2}{2kT_r}\right\} \quad (4)$$

where V_r is the macroscopic drift velocity of the reflecting molecules, T_r the temperature of the reflected molecules, n_r the density of the reflected molecules, and ξ is the velocity of the reflected molecule. The density n_r is obtained by equating the incoming and reflected fluxes, assuming no adsorption or chemical reactions. The geometry of the encounter is illustrated in Fig. 3. It is assumed that V the incoming velocity, V_r the reflected velocity, and the surface normal are in the same plane. This figure also defines the angle of attack δ. Note that $\theta = \pi/2 - \delta$. The model can be simplified by introducing the reflected speed ratio

$$S_r = V_r/(2kT_r/m)^{1/2} \quad (5)$$

This reflected velocity distribution function model has the advantage of being continuous in velocity space, facilitating its aerodynamic application, but the static temperature and drift velocity of the reflected distribution are empirical functions of the incident velocity distribution and its moments and of the nature of the wall. The model reproduces the classical limits of diffuse and specular reflections for appropriate values of the parameters.

Since it has proved difficult to measure T_r, Hurlbut and Sherman[16] introduced the partial thermal accommodation coefficient $\alpha_2(\theta_i)$ to replace the dependence of the model on T_r. Their form of the Nocilla model, called the HSN model,[3] will be used in this work.

Use of the model requires knowledge of the parameters V_r (or S_r), δ_r (or θ_r), and $\alpha_2(\theta_i)$ under the conditions of interest. The partial thermal accommodation coefficient is defined by the equation

$$\alpha_2(\theta_i) = \frac{E_i(\theta_i) - E_r}{E_i(\theta_i) - E_w} \quad (6)$$

where the subscripts have the same meaning as for the other accommodation coefficients. A complete review of the measurements of these parameters is given by Collins.[8] Since it has been much easier to measure S_r than V_r, measurements of V_r will not be discussed.

First, consider the determination of the reflected speed ratio. The earliest determination of the reflected speed ratio was by Nocilla,[17] who fit the thermal energy range data of Hurlbut and Beck[18] to his model. S_r was found to be in the range of 0.75–1.0 with no consistent angular variation. Fujimoto et al.[19] found, from their thermal energy range experiments of Ar and N_2 on Ni, that $S_r = 0.715\theta_i$, θ_i in radians. A number of investigations found reflected speed ratios greater than 1.0, including the work of Cosma et al.,[20] Janda et al.,[21,22] Subbarao and Miller,[23] and Kolodney et al.[24] The latter investigators scattered Hg from MgO in the energy range 1–9 eV, and found supersonic reflection at all scattering angles. Subbarao and Miller[23] scattered Ar from Ag(111). They found that S_r became more difficult to measure as the incident energy increased. At an energy of 1.316 eV, $S_i = 8.2$ and $S_r = 1.5$, but the value of S_r is uncertain. Finally, Gregory and Peters[25] and Karr et al.[26] found $S_r = 0.2$ for a single lobe fit and $S_r = 0.6$ for a double lobe fit from their analysis of a Shuttle Orbiter atomic oxygen scattering experiment. This last experiment, which involved a reactive surface, does not apply to the situation of interest but is included because it is the only scattering experiment that has been performed in orbit. The results of the investigations cited are incomplete, but give the impression that S_r increases with angle from the surface normal and is frequently greater than one in magnitude, especially at large angles. Most of the measurements have been obtained at low energy.

Several measurements have been made of the angle of the mean reflected speed. Nocilla[17] obtained the angle of the mean reflected speed θ_r from the thermal energy data of Hurlbut and Beck.[18] These results can be fit by the equation[8]

$$\theta_r = 1.41\theta_i \quad (7)$$

Fujimoto et al.[19] analyzed their thermal energy range experiments with the Nocilla model and obtained (θ_i, θ_r in radians, T_w in Kelvin) for Ar on Ni

$$\theta_r = 0.530\theta_i - 0.00114T_w + 1.326 \quad (8)$$

and for N_2 on Ni

$$\theta_r = 0.438\theta_i - 0.00072T_w + 1.089 \quad (9)$$

Karr et al.[26] and Gregory and Peters[25] analyzed their Shuttle Orbiter experiments in terms of the equation

$$\theta_r = (\pi/2)P_r + (1 - P_r)\theta_i \quad (10)$$

For a single lobe fit, $P_r = 0.27$; and for a double lobe fit, $P_r = 0.66$. The results cited indicate that θ_r increases with increasing θ_i, but most of these measurements have been obtained at low energy. A review of all scattering experiments[8] indicates that the scattering lobe maximum is never more than a few degrees from the specular angle, and thus Hurlbut and Sherman[16] assumed $\theta_r = \theta_i$.

Considerably more measurements have been made of the thermal accommodation coefficient $\alpha_2(\theta_i)$ than of the other Nocilla model parameters. Some of the more important measurements are summarized in Fig. 4. Included are values obtained indirectly from the momentum accommodation coefficients by Abuaf and Marsden[27] and Varakin and Farafonov[28] and the results of trajectory calculations by Hurlbut[29] for the scattering of 5 eV atomic oxygen from SiO_2. The plotted results span an energy range of 0.17 eV (Ref. 21) to 9 eV (Ref. 24). The conclusion is reached that the thermal accommodation coefficient has a tendency to decrease with increasing θ_i and that its value is most likely considerably less than one; its variation with energy is much less certain.

The measurements discussed have been used to develop several models for the parameters in the HSN model, listed in Table 3. The equation $S_r = S_{r0} + b\sin^n\theta_i$ was suggested by the work of Stark[32] but in no way represents any measurements. The other models represent a fit to at least one

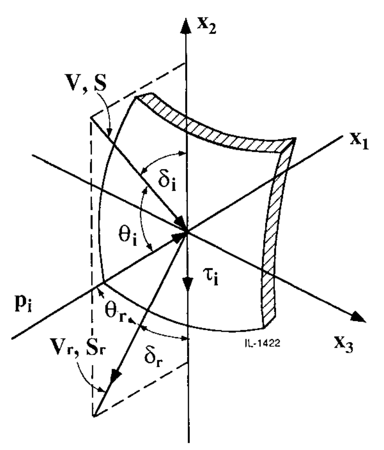

Fig. 3 Surface scattering coordinate system.

Table 3 Empirical constants for the Hurlbut-Sherman-Nocilla model

Set 1 (Hurlbut-Sherman[16])	$S_r = (4/\pi)\theta_i$ $\alpha_2 = 0.6[1 - (2/\pi)\theta_i]$ $\theta_r = \theta_i$
Set A	$\alpha_2 = \begin{cases} 0.607 \cos \theta_i & \theta_i \le 21 \text{ deg} \\ 0.567 \cos(\theta_i - 21 \text{ deg}) & \theta_i > 21 \text{ deg} \end{cases}$ $S_r = 0.5(1 + \sin \theta_i)$ $\theta_i = 0.73\theta_i + 0.424$
Set B	α_2 (same as set A) $S_r = 1 + \sin \theta_i$ θ_r (same as set A)
Set C	$\alpha_2 = \begin{cases} 0.96 & \theta_i \le 35 \text{ deg} \\ 0.96 \cos(\theta_i - 35 \text{ deg}) & \theta_i > 35 \text{ deg} \end{cases}$ $S_r = 0.72\theta_i$ $\theta_r = \theta_i$
Set D	$\alpha_2 =$ (same as set A) $S_r = 0.72\theta_i$ $\theta_r =$ (same as set A)

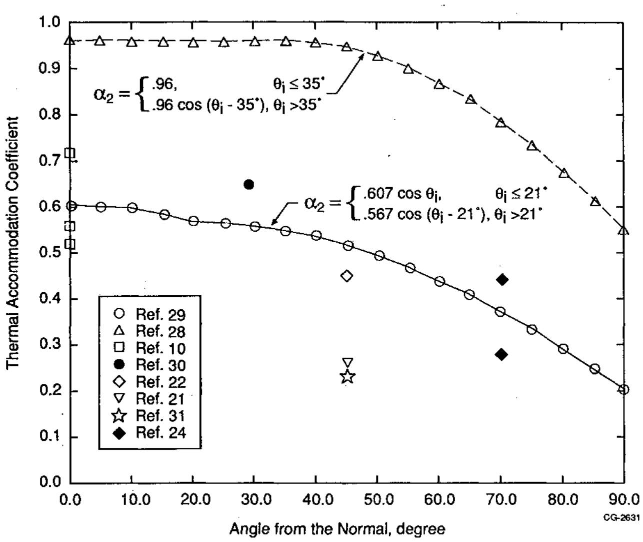

Fig. 4 Measured and computed values of the high energy thermal accommodation coefficient.

data set. Free-molecule aerodynamic coefficients obtained from these models will be compared to those obtained from the accommodation coefficient models in the next section.

Free-Molecule Flat Plate Computations

The accommodation coefficient and HSN models described in the previous sections were used to compute the free-molecule aerodynamic force and moment coefficients for a sphere, sphere-cone, Aeroassist Flight Experiment (AFE) aerobrake, and a flat plate. These results were summarized by Knox et al.[15] and Collins.[8] It was concluded that the flat plate at incidence was the most suitable surface for validating a scattering model. Thus, only flat plate computations will be discussed here. The results reported in this section used the same freestream and body conditions used by Dogra et al.[2] in their calculations at 200 km, namely, $S = 8.7$, $T_w/T = 0.17$, and $\gamma = 1.4$, where T is the freestream temperature and γ the ratio of the specific heats. This freestream condition defines a flow with a well-defined incidence flow direction and is well modeled by high-energy molecular beam scattering experiments.

The free-molecule drag coefficient, lift coefficient, and lift-to-drag ratio computed using the accommodation coefficient and Nocilla models from Tables 2 and 3, respectively, are given in Figs. 5–7. The coefficients are plotted vs δ, the angle of attack. Only the front surface of the plate is included in the calculations. These results are then representative of a small local area on a complex surface.

The drag coefficient predictions fall into three groups, based upon their values at $\delta = 90$ deg. The Nocilla models, with the exception of set C, predict the highest drag coefficient. Set C used the highest value of the thermal accommodation coefficient and resulted in a drag coefficient prediction similar to that of most of the accommodation coefficient models, including diffuse scattering. The exception was fit 16, which predicted a much lower drag coefficient.

A very wide variation of lift coefficients is predicted by the curve fits. Fit 16, which is a fit to the indirectly determined coefficients by Liu et al.,[14] predicts negative lift at all angles of attack, again indicating the difficulty of indirectly determining the accommodation coefficients from flux measurements. Likewise, fit 14, which used a fit to the tangential accommodation coefficient of Doughty and Schaetzle,[13] predicts negative lift at large δ, possibly due to the measured backscattering at large angles.

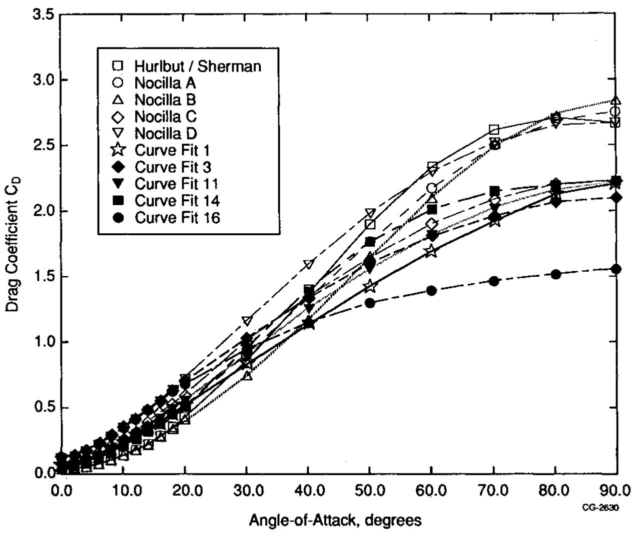

Fig. 5 Computed free-molecule drag coefficient vs angle of attack for a flat plate.

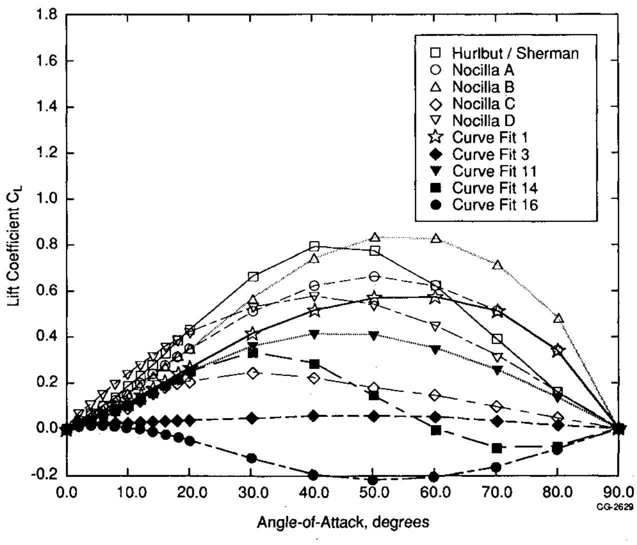

Fig. 6 Computed free-molecule lift coefficient vs angle of attack for a flat plate.

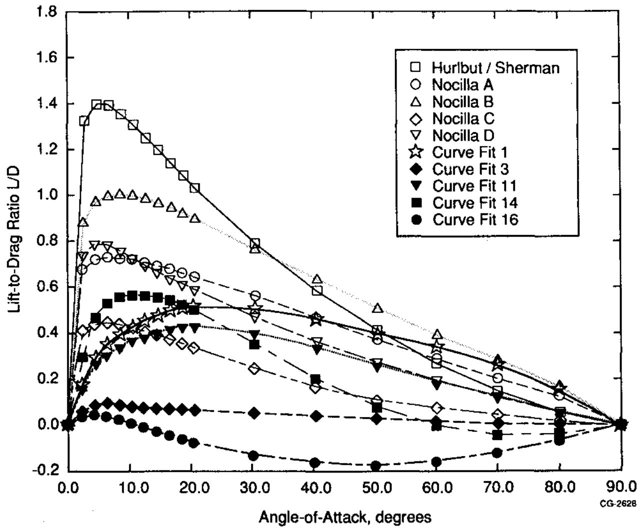

Fig. 7 Computed free-molecule lift-to-drag ratio vs angle of attack for a flat plate.

The lift predictions are more difficult to analyze since they do not follow consistent patterns. However, it can be noted that diffuse scattering (fit 3) predicts lower lift than any other model. That should be compared to the predictions of fit 1, for example. It is fair to assume that the use of diffuse scattering greatly underestimates the amount of lift that can be developed by a vehicle as it proceeds through the atmosphere toward orbit. In general, the Nocilla models predict higher lift coefficients than the accommodation coefficient models, with set C again being the exception.

The lift-to-drag ratio provides a means for classifying the various predictions. The peak value of L/D varies from about 0.09 for diffuse scattering (fit 3) to 1.4 for the Hurlbut-Sherman[16] parameter set, again emphasizing that the traditional use of the diffuse model has probably greatly underpredicted the lift of space vehicles. The Nocilla models predict higher peak values, with the exception of set C. Also, the Nocilla model peaks are at a low angle of attack whereas the accommodation coefficient model predictions peak at a much higher angle. The low angle of the peak value of L/D also occurred for the model introduced by Stark.[32] This behavior of the Nocilla model will be shown to depend on the chosen angular variation of the model parameters and is not an inherent feature of the model.

Analysis of Computations

An attempt will be made to understand the differences between the free-molecule predictions made by the accommodation coefficients and those of the Nocilla model, and to reconcile these differences. It must be recognized that both models are based on incomplete experimental information about the parameters that apply to the conditions experienced by an orbiting vehicle. However, the conditions under which the accommodation coefficient data have been obtained are closer to the orbiting conditions than those for the measurement of the Nocilla model parameters. For purposes of analysis, accommodation coefficient fit 1 will be assumed to be correct. Then the variation of the Nocilla parameters required to give these values of the accommodation coefficients will be determined. Success with this endeavor will then allow the Nocilla model to yield free-molecule aerodynamic coefficients that are identical to those predicted by the accommodation coefficients and will provide a Nocilla model that can be used with the Monte Carlo computational technique to accurately compute rarefied aerodynamic coefficients.

The accommodation coefficients are defined by Eqs. (1) and (2), where now the incident fluxes are taken to depend on θ_i. For a well-defined incident direction, the incident normal and tangential momentum fluxes for free-molecule flow are given by

$$p_i = \rho RT\left\{\frac{\Sigma}{\sqrt{\pi}}\chi(\Sigma) + \frac{1}{2}(1 + \operatorname{erf}\Sigma)\right\} \tag{11}$$

$$\tau_i = \rho RT\,\frac{S\sin\theta_i}{\sqrt{\pi}}\,\chi(\Sigma) \tag{12}$$

where

$$\Sigma = S\cos\theta_i \tag{13}$$

$$\chi(\Sigma) = e^{-\Sigma^2} + \sqrt{\pi}\Sigma(1 + \operatorname{erf}\Sigma) \tag{14}$$

In these equations ρ, R, T, and S represent the density, gas constant, temperature, and speed ratio of the incident stream, respectively.

Using the Nocilla model, the reflected momentum fluxes can be written in an identical fashion, using the reflected properties ρ_r, T_r, θ_r, and S_r.

$$p_r = \rho_r RT_r\left\{\frac{\Sigma_r}{\sqrt{\pi}}\chi(\Sigma_r) + \frac{1}{2}(1 + \operatorname{erf}\Sigma_r)\right\} \tag{15}$$

$$\tau_r = \rho_r RT_r\,\frac{S_r\sin\theta_r}{\sqrt{\pi}}\,\chi(\Sigma_r) \tag{16}$$

where

$$\Sigma_r = S_r\cos\theta_r \tag{17}$$

and $\chi(\Sigma_r)$ is obtained from Eq. (13) by replacing Σ by Σ_r. An artificial density parameter ρ_r is obtained by equating the incident and reflected fluxes. Hurlbut and Sherman[16] have shown that $\rho_r RT_r$ may be written as

$$\rho_r RT_r = (\rho RT)S\sqrt{B}\,\frac{\chi(\Sigma)}{\chi(\Sigma)} \tag{18}$$

where

$$B = \left\{\frac{(1-\alpha_2)\left\{1 + \dfrac{\gamma+1}{1(\gamma-1)S^2} + \dfrac{\sqrt{\pi}\Sigma(1+\operatorname{erf}\Sigma)}{2S^2\chi(\Sigma)}\right\} + \dfrac{\alpha_2}{S^2}\dfrac{T_w}{T}\left[\dfrac{\gamma+1}{2(\gamma-1)}\right]}{S_r^2 + \dfrac{\gamma+1}{2(\gamma-1)} + \dfrac{\sqrt{\pi}\Sigma_r(1+\operatorname{erf}\Sigma_r)}{2\chi(\Sigma_r)}}\right\} \tag{19}$$

The temperature of the reflected distribution is given by the equation

$$(T_r/T) = S^2 B \tag{20}$$

The normal momentum flux re-emitted at the wall temperature is the same as the free-molecule flux emitted by an orifice on a chamber containing gas at the wall temperature and density, where the density is obtained by matching the incident and re-emitted fluxes. The result is

$$p_w = \frac{\rho R T}{2}\left(\frac{T_w}{T}\right)^{1/2}\chi(\Sigma) \tag{21}$$

Equations (11), (12), (15), (16), and (21) can be used in Eqs. (1) and (2) to obtain expressions for the accommodation coefficients. The normal momentum accommodation coefficient expression cannot be simplified, but the tangential momentum accommodation coefficient can be written in the form

$$\sigma = 1 - \sqrt{B}S_r\frac{\sin\theta_r}{\sin\theta_i} \tag{22}$$

As an initial examination of the Nocilla models listed in Table 3, the normal and tangential accommodation coefficients were computed, using the formulas just cited. The following arbitrary parameters were used for the calculations: $S = 9.89$, $T = 974$ K, $T_w = 290$ K, and $\gamma = 1.4$. The resulting computations are compared with those of fit 1, Table 2, in Figs. 8 and 9. All of the models predict a similar angular dependence for the normal accommodation coefficient. However, the Nocilla model predictions become very negative at large values of θ_i. Various tangential accommodation coefficient dependencies are illustrated by the models in Fig. 9. Although the fit 1 data indicate that σ increases with increasing θ_i, this behavior is not predicted by all of the sets of Nocilla parameters. Some of the models provide this angular behavior but predict values that are very negative at low θ_i. It is concluded that none of the proposed Nocilla models predict values of the accommodation coefficients that are close to those that have been measured.

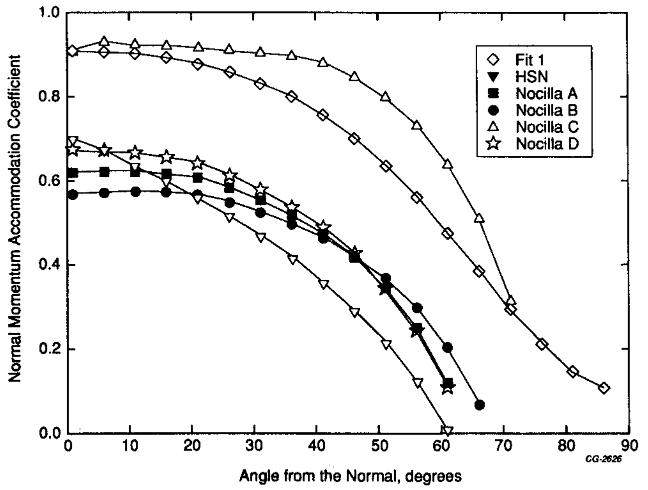

Fig. 8 Normal momentum accommodation coefficients predicted by the Nocilla model.

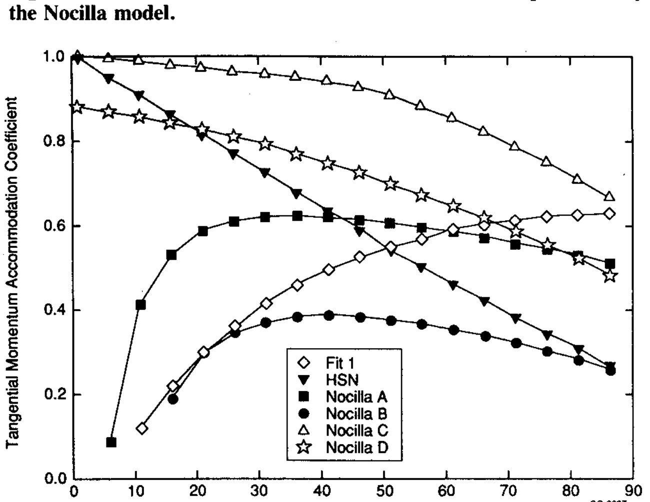

Fig. 9 Tangential momentum accommodation coefficients predicted by the Nocilla model.

These problems with the predicted accommodation coefficients plus the very different lift-to-drag ratio predicted for the flat plate, shown in Fig. 7, are indications that the Nocilla parameter sets given in Table 3 probably are inaccurate. As an additional indication, the parameters S_r and T_r were computed using each set of parameters. The predictions of the various models can be summarized as follows. S_r was always less than 1.0 for set A and reached values greater than 1.0 only for the largest values of θ_i for sets C and D. For the Hurlbut-Sherman model S_r increased from 0 to about 2 as θ_i increased from 0 to 90 deg, and for set B S_r increased from 1 to 2 over the same angular range. The reflected temperature T_r ranged from 9000 K to 23,000 K for all models except set C; for set C T_r varied between 1380 and 9400 K. Although the predictions of the reflected speed ratio could be physically realized, the reflected temperature is not physically possible. It will be shown, however, that these predictions are not an indication of problems with the Nocilla model but with the selection of the parameters used with the model.

The fit 1 accommodation coefficients were assumed to be correct, and an attempt was made to find physically realistic values of the parameters $S_r(\theta_i)$, $\alpha_2(\theta_i)$, and $\theta_r(\theta_i)$ which would yield values of the accommodation coefficients of fit 1. The fit 1 accommodation coefficients are given by the equations

$$\sigma' = 1.0 - 0.9\exp(-2.8\cos^2\theta_i) \tag{23}$$

$$\sigma = 0.9 - 1.2\exp(-1.47\sin^{3/4}\theta_i) \tag{24}$$

The parameters were found by trial and error, using the constraints that $T_r \leq T$ and that the re-emitted flow was not directed into the body surface. The previous discussion indicated that T_r decreased as α_2 increased. Since the previous parameter sets predicted very large values of T_r, values of α_2 close to 1.0 were required to decrease T_r. It is interesting that an exact solution of the preceding equations was not found to exist (convergence was not found after 100 iterations), but a solution giving values of the accommodation coefficients to within 1% of those given by Eqs. (23) and (24) was possible for all θ_i examined. The solutions found are not unique since the accommodation coefficients do not uniquely define the reflected distribution function but a large deviation from the values to be reported is not expected. The values of $S_r(\theta_i)$, $\alpha_2(\theta_i)$, θ_r and $T_r(\theta_i)$ obtained by this process are given in Figs. 10 and 11.

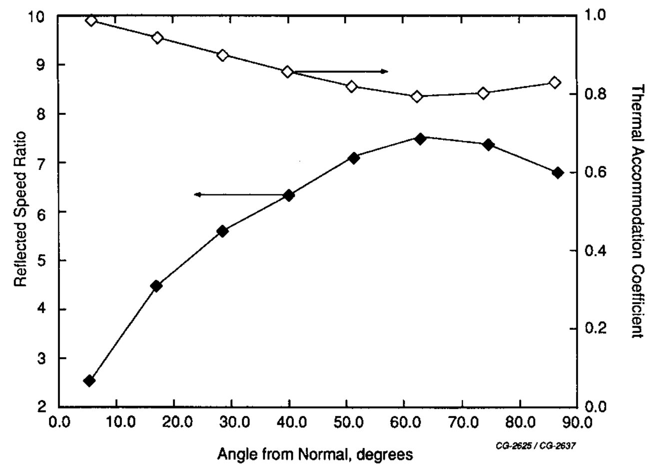

Fig. 10 Variation of reflected speed ratio and thermal accommodation coefficient required for the Nocilla model to reproduce the fit 1 momentum accommodation coefficients.

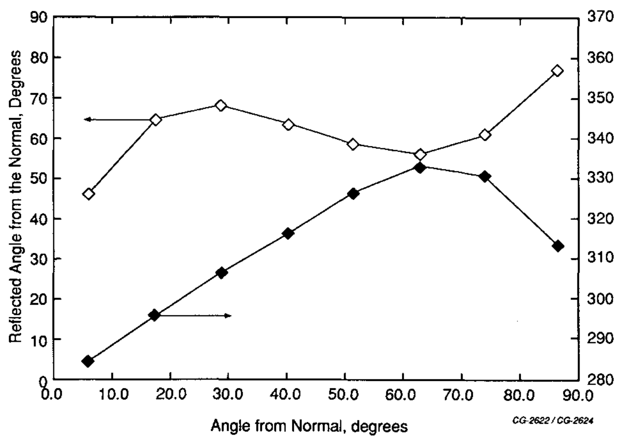

Fig. 11 Variation of the reflected angle and reflected temperature required for the Nocilla model to reproduce the fit 1 momentum accommodation coefficients.

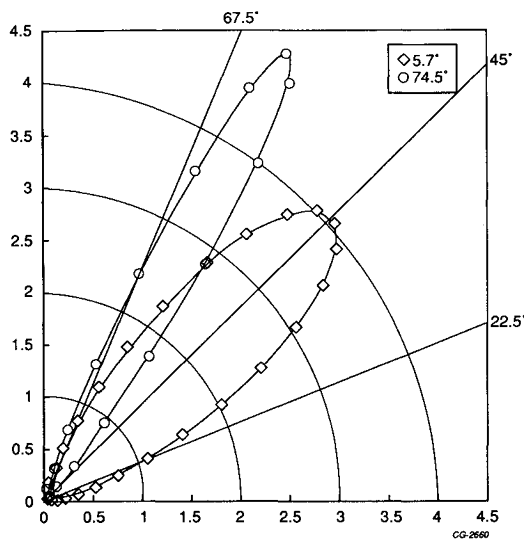

Fig. 12 Reflected flux distributions predicted by the Nocilla model using the parameters which reproduce the model 1 momentum accommodation coefficients.

The magnitudes of some of these parameters can be checked using expressions given by Hurlbut.[3] The maximum value of the partial thermal accommodation coefficient is given by the expression

$$\alpha_{\max} = \frac{[S^2 + (5/2)]T - [S_r^2 T_r + (j/2)T]}{[S^2 + (5/2)]T - [(4 + j)/2]T_w} \tag{25}$$

Comparison of the values shown in Fig. 10 and the value given by Eq. (25), using $j = 2$, indicates that the two are identical to within 1.5%. Notice that the values of $\alpha_2(\theta_i)$ obtained by this process are very similar to that obtained by Varakin and Farafonov,[28] which are represented by the higher values in Fig. 4.

The angle θ_r is not found to vary monotonically with increasing θ_i, Fig. 11. This behavior is explained by an expression for θ_r, derived by Hurlbut[3] based on the assumptions that $S \to \infty$ and that p_w can be neglected. This expression is

$$\theta_r \approx \tan^{-1}\left\{\left(\frac{1 - \sigma'}{1 - \sigma}\right) \tan \theta_i\right\} \tag{26}$$

Comparison of the computed values of θ_r with this expression indicates a 25% deviation at small θ_i but only a 2% deviation at large θ_i. Examination of the magnitude of the individual normal momentum fluxes at small θ_i indicates that p_w is approximately equal to half of p_r and should not have been neglected in Eq. (26) at small angles. Inclusion of p_w explains the observed deviations from Eq. (26). Thus, the nonmonotonic variation of θ_r with θ_i is reasonable for the present situation.

The reflected speed ratio, Fig. 10, is found to increase with increasing θ_i and to always be considerably greater than one. The findings that S_r is greater than one and increases with θ_i are consistent with previous findings.

The temperature of the reflected distribution was found to increase with increasing θ_i, reaching a maximum value at large θ_i (Fig. 11). Almost no data exists with which to compare this prediction.

Re-emitted flux patterns were computed using the equations given by Nocilla.[17] Two representative flux patterns, at $\theta_i = 5.7$ and 74.5 deg, are shown in Fig. 12. The scattering lobes are very narrow. Similar patterns were observed by Hayes et al.[33] for the scattering of Ar from Ag(111) for energy in the range 2.7–18.1 eV. Thus, it is not inconceivable that the patterns could be this narrow.

Conclusions

A limited amount of momentum accommodation coefficient data is available for the conditions met by a vehicle in low Earth orbit. Most of the data have been obtained from direct force measurements, but much of these data have been obtained using ion rather than neutral molecular beams. Use of these data predicts a free-molecule lift coefficient for a flat plate element which is many times greater than that predicted assuming diffuse scattering.

The data set for the Nocilla model parameters under the same orbital conditions is even more limited. Low energy data must be used to obtain a complete parameter set. Use of these parameters yields different predictions for the free-molecule aerodynamic coefficients of a flat plate element than those obtained using the accommodation coefficients. A large lift coefficient is predicted, in agreement with the accommodation coefficient computations, but the lift-to-drag ratio peaks at a much smaller angle. Values of the accommodation coefficients predicted by the Nocilla model do not match the measured values and, in addition, the computed temperature of the reflected distribution is much too large.

It was shown that Nocilla model parameters can be found such that the model yields values of the accommodation coefficients identical to the measured values. These parameters were obtained by trial and error but postanalysis indicates that in the future they can be obtained from accommodation coefficient measurements by using the expressions for $\alpha_{\max}$ and θ_r, Eqs. (25) and (26), derived by Hurlbut.[3] The formula for θ_r should be corrected to account for p_w. Once α_2 and θ_r have been given, S_r can be computed from the accommodation coefficient measurements. The Nocilla parameters found by this process have similar trends to those obtained from low energy experiments. Angle θ_r does not vary monatonically with θ_i, and S_r is much greater than 1.0.

It is concluded that an accurate measurement of the accommodation coefficients under conditions of low Earth orbit can be used to obtain the parameters of the Nocilla model. The Nocilla model then can be used to provide the re-emitted velocity distribution function that is required by the Monte Carlo computational method. Using this re-emitted distribution function, Monte Carlo can be used to obtain accurate values of the aerodynamic coefficients for a vehicle traversing the transition regime. This process requires the accurate determination of the accommodation coefficients. Previous laboratory measurements are not adequate, and the measurements should be obtained by measuring two components of force on a flat plate surface exposed to ambient conditions in orbit.

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